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MIXED ASSEMBLY LINE REBALANCING: A BINARY INTEGER APPROACH APPLIED TO REAL WORLD PROBLEMS IN THE AUTOMOTIVE INDUSTRY

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ABSTRACT−Industrial organizations have increasingly sought to optimize the resources needed for the manufacture of its products from the competition, in order to maintain their profit margins. The search for balance of resources and balanced distribution of tasks in various types of industrial environments is called balancing. When adjustments are made and adequacy of an assembly line that is already in operation, this process is called rebalancing. This paper presents a case study involving a problem of rebalancing of automotive assembly line in an environment of arbitrarily mixed models of products, also known as mix. The proposed procedure for solving the rebalancing in the company in question is based on Binary Integer Programming, in particular the branch and bound algorithm. For comparison, we used a heuristic method based on precedence diagrams for solving the rebalancing of lines. To evaluate the results obtained between the two procedures were used performance indicators such as number of workstations created, average load of work and level of unbalance. The proposed algorithm has resulted in significant improvements in the production line capacity.

KEY WORDS : Balancing and rebalancing of assembly lines, Binary integer programming, Branch, Productivity

1. INTRODUCTION

Increasing competition observed among manufacturing companies, coupled with an increasingly demanding consumer, have forced industrial organizations to undertake changes in their productive system (Tambe, 2006). Currently, the demand for variety and speed delivery of products and services have dominated the market and companies need to adapt to this fact. One way is found to produce several models of a product in the same assembly line in order to satisfy the market with lower delivery times (Souza *et al*., 2003).

This type of production is called product models or simply arbitrarily sequenced mix, which is flexibility, which is a strategic objective and is one of performance indicators for the production function (Slack *et al*., 2001). An assembly line is a stream oriented production system typical of industries for the manufacture of standardized items (Becker and Scholl, 2009).

In an assembly line the tasks are performed for manufacturing of products in a serial manufacturing basis.

The assembly lines mix of product models produce different models arbitrarily mixed, followed by some order, sequence and diversity configuration (Kabir and Tabucanon, 1995). These product lines reflect modern assembly lines, where demand is characterized by high variability, requiring a relatively small volume for each product model. This allows for flexibility and customization in product manufacturing.

Moreover, the environment is more complex due to the variety of products and models of each product that are mounted simultaneously on the line (Corominas *et al*., 2008). The modern demand imposes two types of problem with the product mix: balancing and / or rebalancing of assembly line environment.

These involve the determination of all tasks of all the models for the workstations (Bukchin and Rabinowitch, 2006).

The problem of organizing and assigning tasks to workstations is known for Assembly Line Balancing Problem - ALBP (Bartholdi, 1993). The objective of this problem is to minimize the downtime of the line by reducing the number of stations and or operations or minimizing the cycle time or even a combination of both (Bautista and Pereira, 2009).

Balancing assembly lines is one of the stages of a design process that defines the distribution of tasks according to the physical arrangement, the sequence of construction of the required product (manufacturing schedule) and the necessary resources in order to organize and divide by the transformation stages of the product (Fernandes and Dalalio, 2000).

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The main purpose of an assembly line balancing is to maximize the efficiency and capacity utilization and service demand, delivery and cost required. Thus, a problem of balancing the assembly line is considered to be of industrial importance, because it directly affects the productive performance (Rekiek *et al*., 2002).

When you want to make adjustments and readjustments of an assembly line that is already running, the problem of allocating and organizing tasks to workstations is called assembly line rebalancing (Falknauer, 2005).

In an assembly line, the units under production go from season to season in succession, moving along the line, usually by some sort of conveyor system (Boysen *et al*., 2007).

To Özcan and Toklu (2009), the definition of assembly line is a system in which the units are assembled consecutively in stations connected to each other, turning toward the end of the line, where they usually reach the expected product. While crossing the workstations, the units are manufactured or assembled at a certain time. This time of transformation is known as time of operation that represents the sum of all tasks performed to transform the units produced in a workstation.

The transformation time is limited by the time the crossing of the unit in production, which is the cycle time. At each workstation, tasks are performed by the boundary of the cycle time and these tasks are assigned to stations and organized according to the precedence relations (Özcan and Toklu, 2009).

A precedence is the relationship between the nature of the product where each task a needs to be performed prior to task needs to be performed prior to the task b as well as a task c can only be performed after the completion of a task b, by dependence (Bautista and Pereira, 2007). A precedence diagram is a grouping of tasks represented by a directed graph $G = (V, A)$. Each node i belonging to the set of vertices V of the graph represents a task and each edge of the set of edges that part of the node i has a weight associated with this task, represented by the time required for its implementation.

The direction of the edges represents the dependency ratio and thus the precedence constraint (Boysen *et al*., 2009). The precedence relationships can be represented by diagrams and/or arrays of standard binary integer. Figure 1 is an example of the precedence matrix.

	1	2	3	4	5	6	7	8	9
1		1	0	0	1	1	1	1	1
2			0	0	0	0	1	1	1
3				1	1	1	0	1	1
4					1	1	0	1	1
5						1	0	1	1
6							0	1	1
7								1	1
8									1
9									

Figure 1. Precedence matrix.

An assembly line is balanced or rebalanced when the operation time at each workstation is equal to the cycle time of the assembly line (Fernandes *et al*, 2008).

The cycle time of operation of the line represents the time interval between the output of two consecutive products in a rhythmic line (Askin and Standridge, 1993).

2. PROBLEM OF ASSEMBLY LINE REBALANCING

Consider an industrial environment where there is a variety of products to be produced. For the manufacture of these products, there is a single assembly line. Products can be ordered in lots or in a mix of product models arbitrarily sequenced by customer orders

The allocation of various tasks involved in the manufacture of these products a into a job n consists in a problem of an assembly line rebalancing. It is necessary that each task is assigned to a job and that dependent tasks are set forth in the order required.

It is considered that the job has a known cycle time which limits the allocation of these tasks and that the total time of all tasks assigned to a job, when compared to other jobs, is balanced. Thus, we try to find the smallest number of jobs as possible so that the desired tasks are performed.

The load balancing is a design problem. However, most of the problems of balancing the assembly line is rewriting the existing assembly lines and not the first installation.

It is nothing more than the need to adjust or rearrange an assembly line balancing already existing due to factors such as variations in demand, changes in workload, service to an ergonomic standard, quality requirements for the product in activities and waste disposal, or simply as a strategy to launch the product on the market with lower price and greater flexibility in its construction, therefore, making it competitive.

Conceptually, balancing an assembly line and assembly line rebalancing consist in making the allocation of work over the line according to certain criteria and taking into account certain restrictions (Fernandes and Dalalio, 2000). Of course this problem has more constraints than a problem of balancing.

The problem of balancing an assembly line in an assembly line environment is not of simple polynomial (NP-Hard). The combinatorial nature of this problem makes it difficult to obtain optimal solutions

However, this was the most common environment in industry due to its need to produce various products to achieve customer satisfaction (Gokcen and Erel, 1998).

The problems of balancing the assembly line can be divided into two categories: SALBP - Single Assembly Line Balance Problem and GALBP - General Assembly Line Balance Problem (Diniz, 2005).

The SALBP have been widely researched in recent decades (Karabati and Sayin, 2003). But lately there has been involvement of the scientific community to formulate and solve problems GALBP (Becker and Scholl, 2006).

When a problem of assembly line balancing relax any of the characteristics assumed at SALBP, it falls into the category of GALBP (Fernandes and Dalalio, 2000). The GALBP includes additional restrictions to the problem of balancing single line, such as parallel stations, various provisions of the line of alternative times for the same task, multi-models and / or mix of product model, workstations with two sides, tasks with stochastic times, lines with imbalanced cicles, incompatibility between tasks and space constraints (Becker and Scholl, 2006; Bautista and Pereira, 2007; Boysen *et al*., 2007).

In GALBP, usually every additional feature or characteristic that negates the simplicity (SALBP) of a problem generates a particular variant of the balancing problem. Because of this, Becker and Scholl (2006) argue that this is one reason for the difficulty of establishing criteria for GALBP classification problems.

According to Boysen *et al*. (2007), a classification of problems balancing the assembly line including all relevant goals and constraints, in a structured way, can allow appropriate comparisons. The classification serves to compare the relevant characteristics of the types of problems.

A recent proposal to structure the classification of the problems of balancing the assembly line was made by Boysen *et al*., (2007). This proposal was introduced to denote a tuple notation features and mounting problems, represented as $[\alpha \mid \beta \mid \gamma]$, where α represents the characteristics of the precedence diagram, the characteristics of the β and γ station is the goal (Boysen *et al*., 2008).

2.1. Modeling Problem of Rebalancing

This paper focuses on the rebalancing of an assembly line model mix of products sequenced arbitrarily. Although the issue of balancing is being widely researched in the last 60 years, new specific problems of mix of product models appear, especially given the diversity of environments, assembly lines and specific restrictions for each type of problem balancing / rebalancing (Becker and Scholl, 2006; Agpak and Gokcen, 2007; Boysen *et al*., 2007; Özcan and Toklu, 2009).

The problem of rebalancing the assembly line studied in this paper is based on the classification of Boysen and colleagues type (Boysen *et al*., 2007):

$$reb:[mix, link, inc|\beta|m, SSL^{Line}] \tag{1}$$

Where:

α – Characteristics of the precedence diagram (mix, link, inc)

- *mix*: The models are the product mix;
- *link*: Related tasks that can be fixed in the same station;
- *inc*: Incompatible tasks cannot be combined at the station

β –Characteristics of stations and lines

Line with constant pace, average of work content restricted to the cycle time and global and unique cycle time.

γ – Objectives: $|m$, SSLLine$|$

- Minimize the number of workstations m;
- SSLLine Time of workstations need to be balanced with each other across the board, improving balance, ie, perform the vertical balancing.

To search for a rebalancing solution, the Gokcen and Erel (1998) model was used. In the procedure of Gokcen and Erel (1998), an entire programming model for problems of balancing assembly lines in product mix environments was developed using some properties that allow the reduction of the problem preventing the growth of the variables and therefore limiting the number of constraints.

2.1.1. Formulation of Gokcen and Erel (1998) to the mix of product models

Under the procedure of Gokcen and Erel (1998), the solution to minimizing problem, to reduce variables and consequent simplification of the problem is to decrease the problem of generic rebalancing (GALBP) for simple (SALBP) by applying the combined precedence diagram.

The effort to transform data from a problem of product mix to a problem like GALBP consists in computing the average times of tasks (Thomopoulos, 1970; Macaskill, 1972; Mcmullen and Tarasewich, 2003; Scholl, 1999, cited in Becker and Scholl, 2006). This indicates that, on average, the cycle time will be sufficient to perform all tasks, thereby constituting a SALBP problem. By relaxation definition, similar tasks should be fixed in the same workstation for all product models.

Thus the objective function can be written as:

$$Min \sum_{\kappa=1}^{K} A_{\kappa} \tag{2}$$

Where:

A = Initial number of workstations on which the m model will have tasks to be performed

$k = 1$ if k workstation is used by all models, 0 otherwise;

The objective function of rebalancing presented minimizes the number of workstations used (2), softening the vertical balancing. This means balancing the workload between stations on an assembly line.

With the constraints:

$$\sum_{k=E_i}^{L_i} V_{ik} = 1 \tag{3}$$

Where:

V_{ik} = 1 if the ask i is set for the workstation k; 0, otherwise.

E_i = First Workstation of the task i to be set for a given precedence relation $i = 1,...,N$;

L_i = Last Workstation of the task i that can be set for a given precedence relation $i = 1,...,N$.

- Restriction of Assignment (3): ensures that each type of tasks is assigned to only one workstation.

$$\sum_{k=E_a}^{L_a} k.V_{ak} - \sum_{k=E_b}^{L_b} k.V_{bk} \leq 0 \tag{4}$$

$$\sum_{i \in W_{km}} t_{im}.V_{ik} \leq C_m \tag{5}$$

Where:

t_{im} = task's execution time i of the model m, $m = 1,...,P$;

C_m = ciycles' time of the model m, $m = 1,...,P$.

- Restriction and Precedence (4): In a combined precedence diagram, the relation between the task a and task b, where b is immediately after a.
- Restriction of Cycle Time (5): The sum of the times of the tasks performed for each model must be less than or equal to the cycle time of the model.

$$\sum_{i \in W_{km}} V_{ik} - \|W_{km}\| X_{km} \leq 0 \tag{6}$$

$$\sum_{m=1}^{P} X_{km} - P.A_k = 0 \tag{7}$$

Where:

X_{km} = 1 if the workstation k is used for the model m; 0, if otherwise.

W_{km} = Subgroup of all tasks that can be set on workstation k of model m;

- Restriction Workstation (6): The number of workstations is the same for all models. Even if in a workstation the labor content of the operation for a model equals zero, then the labor content of the workstation for other models will be zero also in that operation.

Where:

i = task;

N = total amount of tasks in the problem;

K = initial amount of workstations;

P = amount of models (products);

PR_i = subgroup of all tasks that precede task i, $i = 1,..., N$;

S_i = subgroup of all tasks that come after task i, $i = 1,..., N$;

t_{im} = execution time of task i of model m, $m = 1,...,P$;

C_m = model m execution's time, $m = 1,...,P$;

E_{im} = First workstation of task i of model m that can be set on a given precedence relation in which i = $1,...,N$; $m = 1,...,P$;

L_{im} = Last workstation of task i of model m that can be set on a given precedence relation in which $i = 1,...,\mathrm{N}$; $m = 1,...,P$;

V_{ik} = 1 if task i is set for a workstation k; 0, otherwise.

X_{km} = 1 if workstation k is used by model m; 0, otherwise.

A_k = 1 if the workstation k is used by all models; 0, otherwise;

W_{km} = Subgroup of all tasks that can be set for workstation k of model m; is a result of E_{im} and L_{im}.

$\|W_{km}\|$ = amount of tasks in the set;

If $X_{km} = 1$ for the workstation k, for $m = 1,...,P$, so $A_k = 1$; otherwise, $A_k = 0$.

According to Gokcen and Erel (1998), before checking the restrictions and objective function, the model requires the definition of first and last stations possible for the tasks in order to reduce the number of variables of the problem.

The first station with task i can be determined based on the fact that the tasks can be spared a sufficient number of stations preceding i. A lower limit in order to determine the first station is the proportion between the time taken to perform the tasks in the model m preceding i plus the execution time of task i in the model m divided by the cycle time of model m, as seen in (8). The same is true in (9), the last station is associated with the tasks posterior to task i in the precedence diagram.

$$E_{im} = \left\lceil \frac{t_m + \sum_{j \in PR_i} t_{jm}}{C_m} \right\rceil^{+} \tag{8}$$

Para $i = 1,...,N$ $m = 1,...,P$

$$L_{im} = K + 1 - \left\lceil \frac{t_{im} + \sum_{j \in S_i} t_{jm}}{C_m} \right\rceil^{+} \tag{9}$$

Para $i = 1,...,N$ $m = 1,...,P$

In order to calculate (9), is necessary to know the value of k. This represents the initial amount of workstations, which is represented in (10) is determined by the sum of the total processing time of all tasks divided by the cycle time of the assembly line for the accomplishment of a unit.

This technique can be used because since the cycle time is known, the number of stations can therefore be empirically estimated. For a known cycle time x, we can allocate the tasks a, b and c since the sum of these time is not greater than the cycle time x. If the sum of all tasks is divided by the cycle time, then a number of stations and can be estimated empirically and comply with the rule cited.

During the elaboration of the initial amount of stations K, one unit is added, for it serves as a margin for the determination of K, since the restrictions of the problem that directly influence the final number of stations were not considered.

$$K = \frac{\sum_{i=1}^{i} t_{im}}{C_m} + 1 \tag{10}$$

Using these concepts presented, and (8), (9) and (10), can reduce the number of variables in the model. Gokcen and Erel applied them to the model mix problem, since the original formulation was made for the simple type. They could use this concept for the reduction of the precedence diagram of each model for a combined precedence diagram.

Typically, there are many tasks that are common among models in product mix assembly lines and similar precedence relations among the tasks in common (Gokcen and Erel, 1998).

Thus, analyzing the common tasks among models, the precedence diagrams of each model can be transformed into one combined precedence diagram. In a combined precedence diagram there should be no conflicts. The precedence diagram reduces the number of variables and restrictions significantly, by sharing common tasks among the models (Gokcen and Erel, 1998). Normally, the number of tasks in a combined precedence diagram is much smaller than the sum of the tasks of the model, due to the fact that common tasks between models appear as a combined task in the diagram.

3. APPLICATION OF THE MODEL IN THE CASE STUDY

The case study affects the final assembly department, where complexity is high due to the amount of needed combinations for building the vehicles.

To determine the rebalancing, the area under study has two key products, which we will call A and B. These products may have three types of engine, with which we shall call 1, 2 and 3. Product A uses only one engine type 1, whereas product B can have the engine 1, 2 or 3. It can be said that the number of combinations is A = B = 1 and 3, total of 4 possible products.

For assembly of the products A and B there are a total of 43 tasks that may or may not be performed on all types of products / engine. There are tasks that affect only one type of engine, while others could affect all models.

Each task has a specific construction time. In addition, all of the tasks have specific times and precedence relations based on technological constraints, layout, quality, safety and / or ergonomics.

In addition to the combinations of products and differences in time and accomplishment of tasks per model, it is necessary to follow the pattern of the tasks according to each quadrant - mounting possibilities. Areas of accomplishment of tasks which create opportunities for assemblies are divided into eight regions of the products, as illustrated in Figure 2.

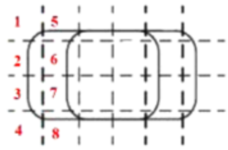

Figure 2. Mounting possibilities of products A and B, adapted from: Becker and Scholl (2009).

The cycle time used as basis was of 72 units of time. The indicators evaluated were of an average work load and of number of workstations created.

3.1. Simulation

The algorithm was applied in MATLAB© version 7.6.0 R2008a (MATHWORKS, 2008). It was initially applied to a small problem with 7 tasks, and then to a problem involving 43 tasks. In the initial application, the cycle time was of 13 units of time and the second application cycle was of 72 units of time.

This means that each new workstation should not have cycle time cycle exceeding these limits of 13 units of time (for the 7 tasks example) or 72 units of time (for the case study, involving 43 tasks).

The methodology BIP-BB (Binary Integer Programming by Branch and Bound) performed showed a satisfactory result when compared with the current configuration used by the company for the rebalancing of the tasks and operations.

Table 1 presents the results obtained through the simulation for generation and creation of new jobs, as well as the level of vertical imbalance or balance or the mildness rate among the operations. Botlenecks are outlined.

The average work load was 93,25% and the imbalance rate was 20,83. The advantage of these results can be seen in the reduction of costs by using the workstations. This may mean that it is possible to perform the tasks by applying fewer resources, because each workstation can use the maximum of its productive capacity, in this case represented by the average work load, showing whether or not there is an opportunity to reduce costs. In the studied model, there was a reduction of costs with the resources

Table 1. Rebalancing results.

Station	Tasks	Time (s)
1	1,2,3,4,5,6,7,8	**71,5**
2	9,10,11,12,13,14,16	70,7
3	15,17,18,19,20,21,22	71,2
4	23,24,25,26,27,28,31,32,36,37	70,9
5	29,30,33,38,41	70,8
6	34,35,39	**53,1**
7	40,42,43	61,8

Table 2. Comparison of time and load between two balancing models.

Gokcen and Erel Model (1998)

Station	Time (s)	Load(%)	Mildness(s)
1	71,50	99,30	0,64
2	70,70	98,20	0,09
3	71,20	98,90	0,36
4	70,90	98,50	0,49
5	70,80	98,30	338,56
6	53,10	73,80	94,09
7	61,80	85,80	
		Total	434,23

Slack Model (1995)

Station	Time (s)	Load(%)	Mildness(s)
1	72,00	100,0	0,96
2	71,02	98,60	0,43
3	71,34	99,10	1,28
4	70,87	98,40	1,97
5	70,59	98,10	367,35
6	52,83	73,40	113,40
7	61,35	85,20	
		Total	485,38

used in the workstations, as it existed prior the rebalancing, 8 workstations in the studied company. The two models presented for solving the rebalancing show the same number of workstations as a solution (7 stations). However, the level of imbalance in the Gokcen and Erel model (1998) shows that the tasks are better distributed among the stations, generating a balance between them, increasing the competitive advantage in this model.

The vertical balancing measures how a line balancing is efficient (Becker and Scholl, 2006). The value obtained through the Mildness or Imbalance Rate (11) gives the variation between the maximum total working time measured among the workstations, and the total working time of the remaining workstations allocated on the production line. The higher the rate, the greater the variation of the total operating time among stations, indicating the low efficiency and the need for a more effective rebalancing (Gerhardt, 2005).

$$Midness = \sqrt{\sum_{k=1}^{\kappa} (t_m - \max - t_{im})^2} \tag{11}$$

It can be seen that the cycle times and loads are changed between the models. The imbalance is smaller in the model adapted from Gokcen and Erel (1998), meaning that the rebalancing is more efficient in this model.

3.2. Model Validation

After applying the model by Gokcen and Erel (1998), the validation of the line rebalancing methodology was made by comparison with the method suggested by Slack *et al.*, 1995.

Slack and his colleagues (Slack *et al.*, 1995) used a heuristic method using precedence diagrams for simple models. An adjustment was made to use this model, by reducing the complexity by using the combined precedence diagram, to be comparable to the model used by Gokcen and Erel (1998).

The comparison was performed using the methods of Slack and the one developed by Gokcen and Erel to improve the balance of station considering some operation. Results confirm that the method proposed by Gokcen and Erel provides comparatively lower operation times results in better output rates than those lines obtained by the Slack method.

The Gokcen and Erel model (1998) has shown better

Table 3. Production capacity for both models.

Gokcen and Erel Model (1998)

	Production capacity (Units)			
Station	Hour	Day	Month	Year
1	50,3	1208,4	36251,7	435021,0
2	50,9	1222,1	36662,0	439943,4
3	50,6	1213,5	36404,5	436853,9
4	50,8	1218,6	36558,5	438702,4
5	50,8	1220,3	36610,2	439322,0
6	67,8	1627,1	48813,6	585762,7
7	58,2	1396,5	41894,3	502731,5

Slack Model (1995)

	Production capacity (Units)			
Station	Hour	Day	Month	Year
1	50,0	1200,0	36000,0	432000,0
2	50,7	1216,5	36496,4	437956,2
3	50,5	1211,0	36330,6	435967,3
4	50,8	1219,1	36574,1	438889,3
5	51,0	1223,9	36716,0	440591,5
6	68,1	1635,3	49059,7	588716,3
7	58,7	1408,3	42248,6	506982,7

results than the presented in Slack *et al*. (1995) to the production. The results for rebalancing has shown that the workstation 1 is a bottleneck. For both models used, this workstation has the longest cycle time. However for Gokcen and Erel model (1998) the workstation 1 is capable of producing 435,021 units against 432,000 units of the model suggested in the Slack *et al*. (1995). The Gokcen and Erel model (1998) can produce 3,021 units more than the Slack model (Slack *et al*., 1995). Table 2 shows results considering imbalance rate where the bottleneck workstation are outlined. Table 3 shows the production capacity for each model considering the production is different windows of time.

4. CONCLUSION

The method by Gokcen and Erel (1998) was applied to solve the problem of balancing of assembly lines in an environment of product models mix used in the automobile industry.

Through this method it was possible to obtain a process model. This model was optimized using the Branch and Bound Algorithm for Binary Integer Programming.

The methodology proposed is illustrated through a case study of the company Auto, of the automotive sector. The algorithm developed was done in MATLAB.

The validation of the algorithm performed was done by comparing the model by Gokcen and Erel (1998) with the model proposed by Slack *et al*. (1995) for line balancing. In both models, the same data were compared to assess the objective of generating the lowest amount possible of workstations, of level of workload and level of imbalance.

The model adapted from Gokcen and Erel (1998) was more efficient than the one from Slack *et al*. (1995), due to significant difference in the imbalance rate.

The simulations show that the results obtained in the model of Gokcen and Erel (1998) show a lower imbalance, which is good, because the main goal of balancing is to distribute the tasks so that all workstations have the cycle time similar as possible among them.

The results also show that the number of outputs in the model of Gokcen and Erel (1998) were 397, while the model presented in the book by Slack shows 412 outputs.

The proposed algorithm was applied in an automotive production line improving the performance. The organization had 8 workstations with 43 tasks and cycle time equal to 72 time unities. After implementing the Gokcen and Erel algorithm it was possible to reduce the number of workstations from 8 to 7 realizing the same number of tasks with the same cycle time. Furthermore, the imbalance rate was improved to from 27,48 to 20,83.

To produce 1000 unities in a given period of time using the proposed algorithm it is possible to reduce costs in the workstations. Comparing costs of the previous production line and the one implemented with the new algorithm it was possible to reduce the cost up to 12,5%.

As the imbalance provides the variation between the maximum total working time measured among the workstations and the total working time of the remaining workstations allocated on the production line, the higher the rate, the worse the balance, indicating a need of adjustment.

Therefore, the results presented in model by Gokcen and Erel (1998) are better than the one by Slack *et al*. (1995). The result of the optimization showed an improved performance for industrial activity.

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